FACTORIZATION
in semi-inclusive processes: tmd approach

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- factorization in semi-inclusive processes within the tmd approach

- extra divergences of the transverse-momentum dependent (tmd) parton densities and their influence on the renormalization properties

- generalized definition of the tmd pdf, consistent with the factorization

- a set of the evolution equations for tmd pdf

KINEMATICS

$$\gamma^*(q) + H_1(P) \to \mathcal{X} + H_2(P')$$

$$l^\mu = (l^+, l^-, \boldsymbol{l}_\perp)\ ,\ l^\pm = (l^0 \pm l^3)/\sqrt{2}\ ,\ l^2 = 2l^+l^- - \boldsymbol{l}_\perp^2$$

$$n^{*\mu} = \Omega(1, 1, \mathbf{0}_\perp)\ ,\quad n^\mu = \frac{1}{2\Omega}(1, -1, \mathbf{0}_\perp)\quad ,\quad n^{*+} = \sqrt{2}\Omega$$

$$n^{*-} = 0\ ,\ n^+ = 0\ ,\ n^- = \frac{1}{\sqrt{2}\Omega}\ ,\quad n^* n = 1\ ,\ (n^*)^2 = n^2 = 0$$

$$P^\mu = n^{*\mu} + \frac{M^2}{2} n^\mu\ ,\quad P^2 = M^2$$

$$q^\mu = -x_N n^{*\mu} + \frac{Q^2}{2x_N} n^\mu \quad \to \quad q^+ = -\sqrt{2} x_N \Omega\ ,\quad q^- = \frac{Q^2}{2\sqrt{2} x_N \Omega}$$

x_N — Nachtmann variable

$x_B = Q^2/2(Pq)$ — Bjorken variable

$$\sqrt{2}\Omega = P^+ \to x_B = \frac{x_N}{1 - \frac{M^2}{Q^2}x_N^2} = x_N + O\left(\frac{M^2}{Q^2}\right)$$

kinematical approximations are important!

Collins, Rogers, Stasto: PRD (2008)

→ **fully unintegrated** parton correlation functions

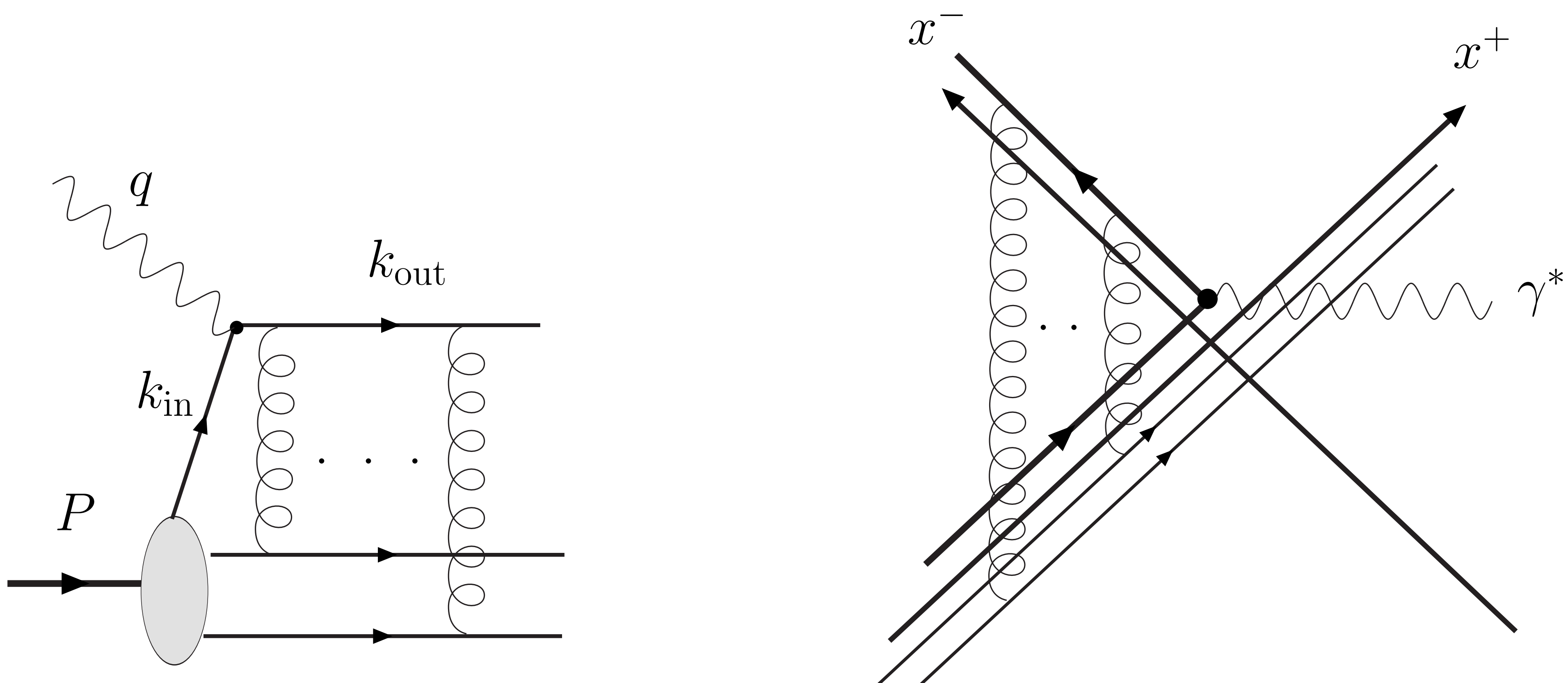
M^2/Q^2 corrections neglected

$$x_B \approx x_N$$

$$P^\mu = \left(P^+, \frac{M^2}{2P^+}, \mathbf{0}_\perp\right) \quad , \quad q^\mu = \left(-x_B P^+, \frac{Q^2}{2x_B P^+}, \mathbf{0}_\perp\right)$$

$$P^+ \sim E_P = \text{hadron energy}$$

$$s \sim \frac{Q^2}{x_B}$$



(a) (b)

INCLUSIVE PROCESSES (DIS)

hadronic tensor

$$W_{\mu\nu} = \frac{1}{2\pi} \Im m \left[i \int d^4\xi \, \mathrm{e}^{iq\xi} \langle P | T\{J_\mu(\xi) J_\nu(0)\} | P \rangle \right]$$

for unpolarized target

$$W_{\mu\nu} =$$

$$= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) F_1(x_B, Q^2) + \frac{1}{P \cdot q} \left(P_\mu - q_\mu \frac{P \cdot q}{q^2} \right) \left(P_\nu - q_\nu \frac{P \cdot q}{q^2} \right) F_2(x_B, Q^2)$$

FACTORIZATION

DIS

$$\gamma^*(q) + \mathbf{H_1}(\mathbf{P}) \to \mathcal{X}$$

$$F(x_B, Q^2) = H(x_B, Q^2/\mu^2) \otimes F_D(\mu^2)$$

$$= \sum_i \int_{x_B}^1 \frac{d\xi}{\xi} C_i\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}\right) F_D^i(\xi, \mu^2)$$

$$F_1(x_\mathrm{B}, Q^2) = \frac{1}{2x_\mathrm{B}} F_2\left(x_\mathrm{B}, Q^2\right) = \frac{1}{2}\sum_i e_i^2 \left[Q_i(x_\mathrm{B}, Q^2) + \bar{Q}_i(x_\mathrm{B}, Q^2)\right]$$

integrated parton densities:

definition; gauge invariance; RG properties

quark distribution:

$$Q'_{i/h}(x,\mu) = \frac{1}{2}\int \frac{d\xi^-}{2\pi}\, \mathrm{e}^{-ik^+\xi^-} \langle h(P)|\bar{\psi}_i(\xi^-, \mathbf{0}_\perp)\gamma^+\psi_i(0^-, \mathbf{0}_\perp)|h(P)\rangle \sim$$

$$\sim \langle h(P)|a_i^\dagger a_i(x)|h(P)\rangle$$

gauge invariance: insertion of the **gauge link**

$$[y, x|\Gamma] = \mathcal{P}\exp\left[-ig\int_{x[\Gamma]}^{y} dz_\mu A_a^\mu(z) t_a\right]_{\mathrm{F}}$$

completely **gauge invariant** (quark) density:

$$Q_{i/h}(x,\mu) = \frac{1}{2}\int \frac{d\xi^-}{2\pi}\, \mathrm{e}^{-ik^+\xi^-} \langle h(P)|\bar{\psi}_i(\xi^-,\mathbf{0}_\perp)[\xi^-,0^-]\gamma^+\psi_i(0^-,\mathbf{0}_\perp)|h(P)\rangle$$

renormalization group properties: **DGLAP**

$$\mu\frac{d}{d\mu}\mathcal{P}_{i/h}(x,\mu) = \sum_j \int_x^1 \frac{dz}{z}\, P_{ij}\left(\frac{x}{z}\right) \mathcal{P}_{j/h}(x,\mu)$$

$$\hat{\mathcal{P}}_{i/h}(x,\mu) \to \hat{\mathcal{P}}_{i/h}(x,Q^2)$$

SEMI – INCLUSIVE PROCESSES

$$\gamma^*(q) + \mathbf{H_1}(\mathbf{P}) \rightarrow \mathbf{H_2}(\mathbf{P}') + \mathcal{X}$$

approaches to **semi-inclusive DIS**

large $\boldsymbol{P}_\perp$ $\boldsymbol{P}_\perp \sim Q$
large Q^2 $Q^2 \gg \Lambda^2_{\mathrm{QCD}}$

perturbative calculations with **integrated** densities

Meng, Olness, Soper: NPB (1992)

moderate $\boldsymbol{P}_\perp$ $\Lambda_{\mathrm{QCD}} \ll \boldsymbol{P}_\perp \ll Q$
large Q^2 $Q^2 \gg \Lambda^2_{\mathrm{QCD}}$

perturbative calculations with **integrated** densities plus resummation of large double logs $\alpha_s \ln^2 \boldsymbol{P}_\perp / Q$

Collins, Soper: NPB (1981, 1982)

Dokshitzer, Diakonov, Troian: PR (1980) *et al.*

FACTORIZATION at small $\boldsymbol{P}_\perp$ — SIDIS

Ji, Ma, Yuan: PRD (2005)

small $\boldsymbol{P}_\perp$ — $\boldsymbol{P}_\perp \sim \Lambda_{\rm QCD}$

moderate Q^2 — $Q^2 \sim 10^2\ {\rm GeV}^2$

$$F(x_B, z_h, \boldsymbol{P}_{h\perp}, Q^2) = \sum_i e_i^2 \cdot$$

$$\cdot H(Q^2, \mu^2, \rho) \otimes \mathcal{F}_D(x_B, \boldsymbol{k}_\perp, \mu^2, x_B\zeta, \rho) \otimes \mathcal{F}_F(z_h, \boldsymbol{q}_\perp, \mu^2, \hat{\zeta}/z_h, \rho) \otimes S(\boldsymbol{l}_\perp^2, \mu^2, \rho)$$

$$\zeta^2 x_B^2 = \frac{\hat{\zeta}^2}{z_h^2} = Q^2 \rho$$

μ = renormalization (collinear factorization) scale

ρ = rapidity cutoff

“naive definition of **tmd density**

$$\mathcal{Q}^{?}(x, \boldsymbol{k}_\perp) = \frac{1}{2} \int \frac{d\xi^- d^2\xi_\perp}{2\pi(2\pi)^2} \; \mathrm{e}^{-ik^+\xi^- + ik_\perp \cdot \xi_\perp}.$$

$$\cdot \langle P | \bar{\psi}_i(\xi^-, \xi_\perp) [\xi^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\xi}_\perp;]^\dagger \gamma^+ [\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp] \psi_i(0^-, 0_\perp) | P \rangle \,|_{\xi^+=0}$$

structure of the **gauge links** is much more complicated, than in the **integrated** case; yields additional problems for factorization in different gauges.

divergences in the **tmd** case

- usual **UV-singularities**, which can be removed using the standard $R-$operation: controlled by the UV-evolution, e.g., in the fully integrated case it yields DGLAP equation.

- **rapidity divergences**, which appear only in the **unintegrated** case: cancel in the integrated distributions, but in the **tmd** case they remain and affect the structure of logarithmic terms to be resumed by a consistent procedure. source: uncompensated light-cone artifacts, which stems from either the light-like gauge links (in the covariant gauges), of from the additional terms in the gluon propagator in the (singular) light-cone axial gauge.

- **mixed divergences**: contain both UV and rapidity singularities simultaneously. highly undesirable, since they break the correct UV-evolution, and depend on the parameters of the chosen gauge, what makes the definition of TMD PDF invalid from the point of view of the complete gauge invariance.

avoid “rapidity problems:

- **shift from the light cone** or use of non-light-like axial gauge (Collins, Soper: NPB (1981, 1982))
- **subtraction** of the soft factor (Collins, Hautmann: PLB (2000), JHEP (2001))
- **regularization** of the light-cone gauge (Cherednikov, Stefanis: PRD (2008, 2009), NPB (2008))

(at the momemnt) **factorization** is demonstrated only within the first approach

shift from the light cone in the Feynman gauge:

$$\mathcal{Q}^{\mathrm{Feynman}}(x, \boldsymbol{k}_\perp) = \frac{1}{2} \int \frac{d\xi^- d^2\xi_\perp}{2\pi(2\pi)^2} \; \mathrm{e}^{-ik^+\xi^- + ik_\perp \cdot \xi_\perp}.$$

$$\cdot \langle p | \bar{\psi}_i(\xi^-, \xi_\perp) [\xi^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\xi}_\perp;]_v^\dagger \gamma^+ [\infty^-, \boldsymbol{0}_\perp; 0^-, \boldsymbol{0}_\perp]_v \psi_i(0^-, 0_\perp) | p \rangle \; |_{\xi^+=0}$$

arbitrary vector v introduces new dimensional variable

$$\zeta = \frac{(2P \cdot v)^2}{v^2}$$

regularization of the gluon propagator in the light-cone gauge:

$$d^{\mu\nu}(q) = -(q^\mu n^{-\nu} + q^\nu n^{-\mu})\frac{1}{2}\left(\frac{1}{q^+ + i\eta} + \frac{1}{q^+ - i\eta}\right)$$

another dimensional variable η

always stay "**on the light-cone**; naturally reproduce **integrated** case (DGLAP); easier (?) derive **rapidity evolution**

PROOF of FACTORIZATION in the covariant (Feynman) gauge

Ji, Ma, Yuan: PRD (2005), PLB (2004); Idilbi & JMY: PRD (2004)

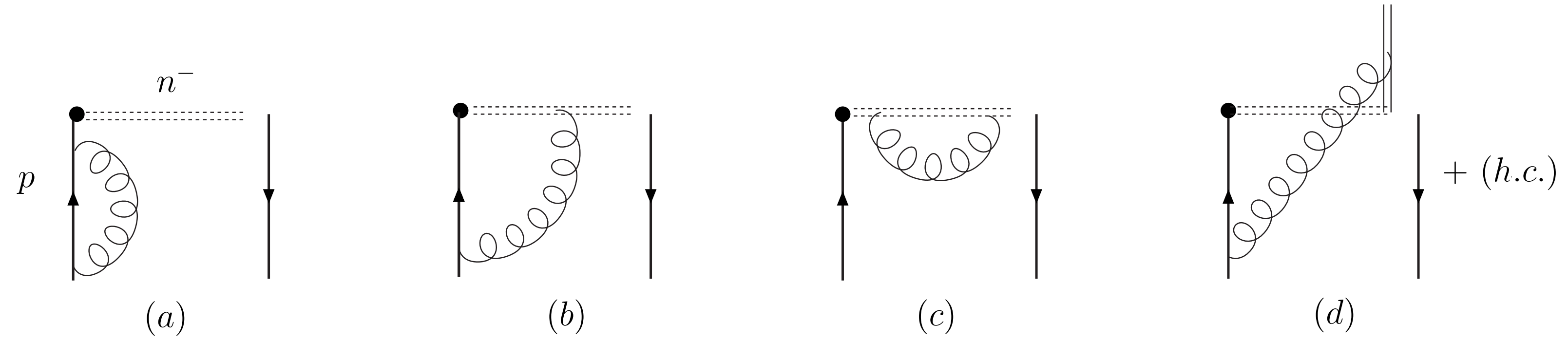
extra dimensionless variable: rapidity cutoff ρ

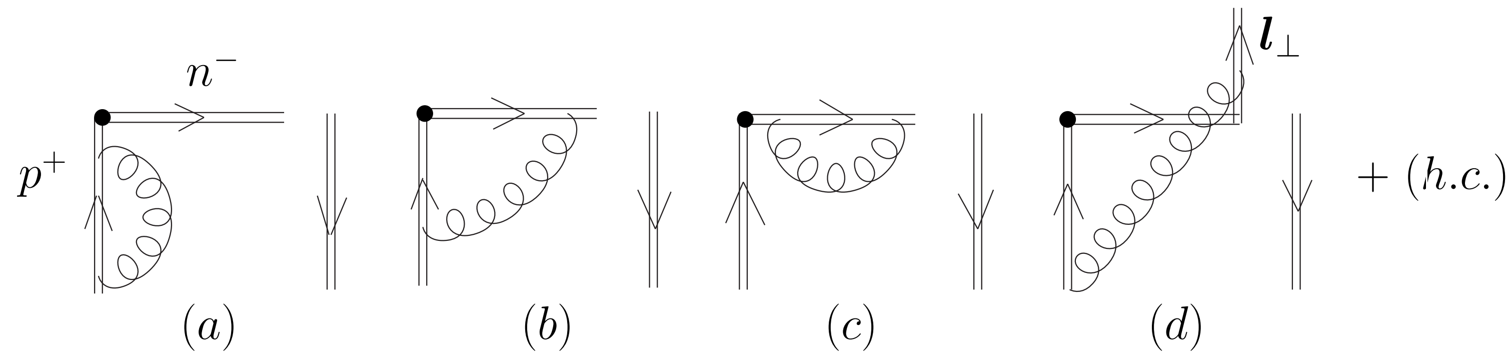
$$\rho = \sqrt{\frac{v^- u^+}{v^+ u^-}}$$

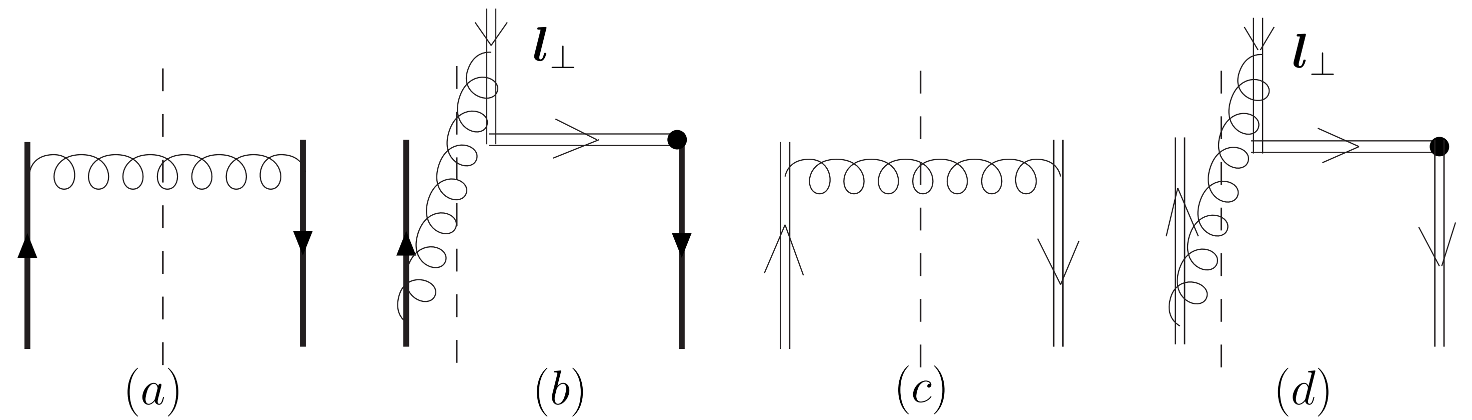
soft factor

$$S(\boldsymbol{\xi}_\perp, \mu^2, \rho) = \langle 0 | [\xi, -\infty]_u^\dagger \, [\infty, \xi]_v^\dagger [\infty, 0]_v [0, -\infty]_u | 0 \rangle$$

calculation of the **one-gluon** diagrams







factorized structure function is demonstrated to be ρ- and μ-**independent**

a set of **evolution equations** for unintegrated densities

- **UV-evolution** (in the integrated case—DGLAP)

$$\mu\frac{d}{d\mu}\,\mathcal{P}(x,\boldsymbol{k}_\perp,\mu,\zeta) = \mathcal{K}_{\rm UV}\otimes\mathcal{P}(x,\boldsymbol{k}_\perp,\mu,\zeta)$$

- **rapidity evolution** (Collins-Soper equation) (no correspondence in the integrated case!)

$$\zeta\frac{d}{d\zeta}\,\mathcal{P}(x,\boldsymbol{k}_\perp,\mu,\zeta) = \mathcal{K}_{\rm CS}\otimes\mathcal{P}(x,\boldsymbol{k}_\perp,\mu,\zeta)$$

- **BFKL evolution** (relation to the Collins-Soper evolution is not known!)

$$x\frac{d}{dx}\,\mathcal{P}(x,\boldsymbol{k}_\perp,\mu,\zeta) = \mathcal{K}_{\rm BFKL}\otimes\mathcal{P}(x,\boldsymbol{k}_\perp,\mu,\zeta)$$

UV-evolution

$$\frac{1}{2}\,\mu\frac{d}{d\mu}\,\mathcal{Q}(x,\boldsymbol{k}_\perp) = \int d^2\boldsymbol{q}_\perp\,\int_x^1\frac{dz}{z}\,P_\perp\left(\frac{x}{z},\boldsymbol{q}_\perp,\alpha_s\right)\,\mathcal{Q}(z,\boldsymbol{q}_\perp)$$

$$P_\perp\,(y,\boldsymbol{q}_\perp,\alpha_s) = \gamma\;\delta(1-y)\;\delta^{(2)}(\boldsymbol{k}_\perp-\boldsymbol{q}_\perp) + O(\alpha_s^2)\;,$$

$$\gamma = \gamma_{2q} = \frac{3}{4}\frac{\alpha_s}{\pi}\,C_{\rm F} + O(\alpha_s^2)\;.$$

rapidity evolution

$$\zeta \frac{\partial}{\partial \zeta} \mathcal{Q}(x, \boldsymbol{k}_\perp, \mu, \zeta) =$$

$$= \frac{\alpha_s C_F}{\pi} \int d^2 q_\perp \left[\left(1 - \ln \frac{x^2 \zeta^2}{\mu^2} \right) \delta^{(2)}(\boldsymbol{q}_\perp - \boldsymbol{k}_\perp) + \frac{1}{2\pi} \frac{1}{(\boldsymbol{k}_\perp - \boldsymbol{q}_\perp)^2 + \lambda^2} \right] \mathcal{Q}(x, \boldsymbol{q}_\perp, \mu, \zeta)$$

(preliminary?) conclusions

- within the **tmd** approach, factorization of semi-inclusive DIS and Drell-Yan processes is demonstrated
- results in the **light-cone gauge** are still missing
- complete set of **evolution equation** is not (?) known
- reduction to the **integrated** case: questionable (at least, in the covariant gauges)
- work is (always) in progress :)